1. Solve the system by addition: $\begin{cases} 3x-2y=6 \\ 2x+4y=20 \end{cases}$

Solution:

Multiply the first equation by 2 and add the corresponding terms:

$$\begin{array}{r} +\begin{cases} 6x-4y=12 \\ 2x+4y=20 \end{cases} \\ \hline 8x=32 \end{array}$$

Solve for x, dividing both sides by 8: $x=4$.

Plug in any of the equations above $x=4$ to find the y value:

$$3x-2y=6$$
$$3(4)-2y=6$$
$$12-2y=6$$
$$\cancel{12}-2y\cancel{-12}=6-12$$
$$-2y=-6$$

Divide both sides by -2: $y=3$.

The solution is: $\boxed{(4,3)}$.

Check your solution by plugging in the values in BOTH equations!

2. Solve the system using the substitution method: $\begin{cases} 7x-y=24 \\ x=2y+9 \end{cases}$

Solution:

The second equation is solved for x. Plug in the second equation into the first equation:

$$7(2y+9)-y=24$$

Follow the order of operation to solve for y:

$$14y+63-y=24$$
$$13y+63=24$$
$$13y\cancel{+63}\cancel{-63}=24-63$$
$$13y=-39$$

Divide by 13 both sides to solve for y: $y=-3$.

Plug in any of the equations above $y=-3$ to find the x value:

$x=2(-3)+9$

$x=-6+9$

$x=3$

The solution is: $\boxed{(3,-3)}$.

Check your solution by plugging in the values in BOTH equations!

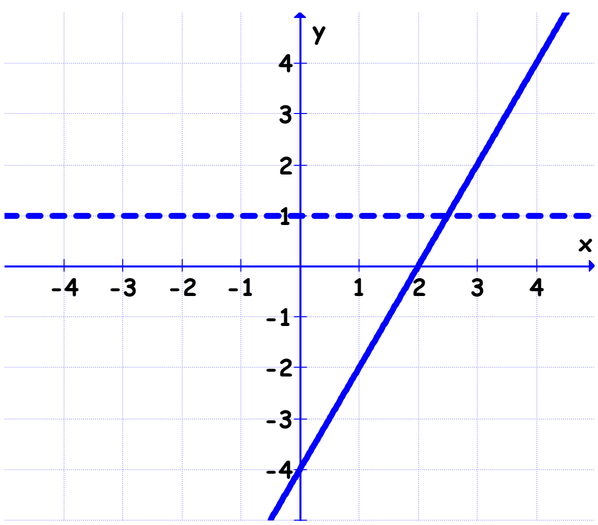
3. Solve the system by graphing: $\begin{cases} 2x-y \geq 4 \\ y>1 \end{cases}$

Solution:

Graph the system of equations:

$\begin{cases} 2x-y=4 \\ y=1 \end{cases}$

The first inequality has to be graphed with a bold line. The second inequality is a strong inequality, so it has to be graphed with a dotted line.

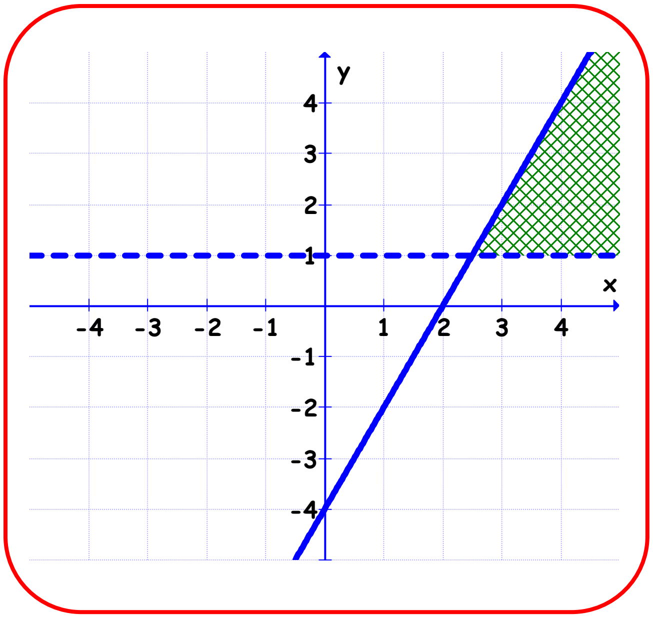


Perform a point check. Take a point from INSIDE of the areas and plug in in the INEQUALITIES!

I will pick the point $(0,0)$

$\begin{cases} 2(0)-0 \geq 4 \quad \textit{wrong} \\ 0>1 \quad \textit{wrong} \end{cases} \Rightarrow$ The point $(0,0)$ DOES NOT belong to the solution!

The solution is:



4. a) If y varies directly as x, find the constant of variation and the direct variation equation for this situation: $y = 26$, when $x = 2$.

Solution: Direct variation looks like y = kx. To find k = ? we substitute the given information:

$$y = kx$$
$$26 = 2k$$
$$\frac{26}{2} = \frac{2k}{2}$$
$$\boxed{k = 13 \quad \Rightarrow \quad y = 13x}$$

b) If y varies inversely as x, find the constant of variation and the direct variation equation for this situation: $y = 5$ when $x = 4$.

Solution: Direct variation looks like $y = \frac{k}{x}$. To find k = ? we substitute the given information:

$$y = \frac{k}{x}$$
$$5 = \frac{k}{4}$$
$$5(4) = k$$
$$\boxed{k = 20 \quad \Rightarrow \quad y = \frac{20}{x}}$$

Graph the solution on a real number line and write the solution in interval notation.

5. $|x-1|+4<5$

Solution:

Solve for the absolute value:

$|x-1|\cancel{+4}\cancel{-4}<5-4$

$|x-1|<1$

There are many ways to be thought about this kind of problems. You are free to use whatever you have learned best. I am going to use the definition for absolute value:

$|z|=\pm z$:

$|x-1|<1$

$\pm(x-1)<1$

This yields 2 problems:

$x-1<1$

$x-1+1<1+1$

$x<2$

$-(x-1)<1$

$-x+1<1$

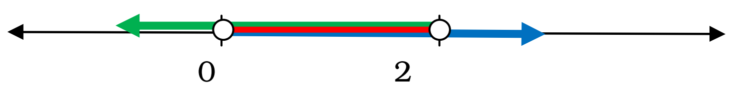
$-x\cancel{+1}\cancel{-1}<1-1$

$-x<0$

$x>0$

Multiply both sides with (– 1). REMEMBER to inverse the direction of the inequality when you multiply with a negative number!

The solution is: {x/x<2, x>0}. In interval notation: (0, 2). Graphically:



6. $\left|2x-4\right|\geq 8$

Solution:

The equation is already solved for the absolute value. I am going to use the strategy from above to solve the inequality:

$$\left|2x-4\right|\geq 8$$

$$\pm(2x-4)\geq 8$$

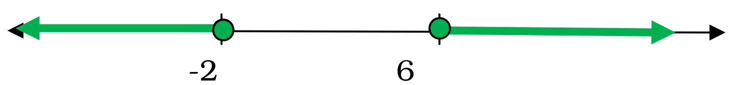
This yields 2 problems:

$$\begin{aligned}&2x-4\geq 8\\&2x-4+4\geq 8+4\\&2x\geq 12\\&x\geq 6\end{aligned}$$

$$\begin{aligned}&-(2x-4)\geq 8\\&-2x+4\geq 8\\&-2x+4-4\geq 8-4\\&-2x\geq 4\\&x\leq -2\end{aligned}$$

REMEMBER to inverse the direction of the inequality when you divide by a negative number!

The solution is: {x/x≤-2, x≥6}. In interval notation: (-∞, -2]∪[6, ∞). Graphically:



7. Solve for x: $\left|3x-1\right|-5=3$

Solution:

Solve for the absolute value: $\left|3x-1\right|-5+5=3+5$

$$\left|3x-1\right|=8$$

$$\pm(3x-1)=8$$

This yields 2 problems:

$$\begin{aligned}&3x-1=8\\&3x-1+1=8+1\\&3x=9\\&x=3\end{aligned}$$

$$\begin{aligned}&-(3x-1)=8\\&-3x+1=8\\&-3x+1-1=8-1\\&-3x=7\\&x=-\frac{7}{3}\end{aligned}$$

The solution is: $\boxed{x=3, x=-\frac{7}{3}}$

8. Using function notation, write the equation of the line with slope of 2, which passes through P (1, 3).

Solution:

The equation of a line in point-slope form is:

$$y - y_1 = m(x - x_1)$$
$$x_1 = 1; y_1 = 3; m = 2$$
$$y - 3 = 2(x - 1)$$

Following the order of operations:

$$y - 3 = 2x - 2$$
$$y - 3 + 3 = 2x - 2 + 3$$
$$y = 2x + 1$$
$$\boxed{f(x) = 2x + 1}$$

9. Using function notation, write the equation of the line with a y intercept point of (0, 2) and is perpendicular to $\frac{1}{3}x + y = 5$.

Solution: The slope of the perpendicular line is reciprocal and opposite of the slope of the line $\frac{1}{3}x + y = 5$. Solve for y this equation to find the slope:

$$\frac{1}{3}x + y = 5$$
$$y = -\frac{1}{3}x + 5$$

The slope of the perpendicular line is $m = -\frac{1}{3}$. The slope of the line that is required in this problem is $m = 3$.

Using the point-slope formula: $y - y_1 = m(x - x_1)$

$$y - 2 = 3(x - 0)$$
$$y - 2 = 3x$$
$$y = 3x + 2$$
$$\boxed{f(x) = 3x + 2}$$

In 10 and 11, simplify each expression. Write answers without using negative exponents.

10. $\left(\dfrac{-x^3}{27y^{-6}}\right)^{\frac{2}{3}}$

Solution: $\left(\dfrac{-x^3}{27y^{-6}}\right)^{\frac{2}{3}} = \dfrac{\left(-x^3\right)^{\frac{2}{3}}}{(27)^{\frac{2}{3}}\left(y^{-6}\right)^{\frac{2}{3}}} = \dfrac{(-x)^{3\left(\frac{2}{3}\right)}}{(27)^{\frac{2}{3}}\, y^{-6\left(\frac{2}{3}\right)}} = \dfrac{(-x)^2}{\left(\sqrt[3]{27}\right)^2 y^{-4}} = \dfrac{x^2}{3^2 y^{-4}} = \boxed{\dfrac{x^2y^4}{9}}$

11. $64^{\frac{-1}{2}}$

Solution: $64^{\frac{-1}{2}} = \dfrac{1}{64^{\frac{1}{2}}} = \dfrac{1}{\sqrt{64}} = \boxed{\dfrac{1}{8}}$

In 12-15, simplify each expression. Assume that all variables represent positive real numbers. Add or subtract like terms where indicated.

12. $\sqrt{49x^6y^{16}z^4}$

Solution: $\sqrt{49x^6y^{16}z^4} = \sqrt{49}\sqrt{x^6}\sqrt{y^{16}}\sqrt{z^4} = 7x^{\frac{6}{2}}y^{\frac{16}{2}}z^{\frac{4}{2}} = \boxed{7x^3y^8z^2}$

13. $\dfrac{\sqrt[3]{48x^8y^{14}}}{\sqrt[3]{2x^2y^2}}$

Solution: $\dfrac{\sqrt[3]{48x^8y^{14}}}{\sqrt[3]{2x^2y^2}} = \sqrt[3]{\dfrac{48x^8y^{14}}{2x^2y^2}} = \sqrt[3]{24x^{8-2}y^{14-2}} = \sqrt[3]{24x^6y^{12}} = \sqrt[3]{24}\sqrt[3]{x^6}\sqrt[3]{y^{12}} =$

$= \sqrt[3]{2^3\cdot 3}x^{\frac{6}{3}}y^{\frac{12}{3}} = 2\sqrt[3]{3}x^2y^4 = \boxed{2x^2y^4\sqrt[3]{3}}$

14. $\sqrt{27} - \sqrt{48}$

Solution: $\sqrt{27} - \sqrt{48} = \sqrt{3^2\cdot 3} - \sqrt{4^2\cdot 3} = 3\sqrt{3} - 4\sqrt{3} = \boxed{-\sqrt{3}}$

15. $\sqrt[3]{32x^4} + 6x\sqrt[3]{4x}$

Solution: $\sqrt[3]{32x^4} + 6x\sqrt[3]{4x} = \sqrt[3]{2^3\cdot 4\cdot x^3\cdot x} + 6x\sqrt[3]{4x} = 2x\sqrt[3]{4x} + 6x\sqrt[3]{4x} =$

$\boxed{= 8x\sqrt[3]{4x}}$

In 16-18, multiply or divide as indicated. Rationalize all denominators.

16. $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)$

Solution: $\left(4\sqrt{3}-3\sqrt{2}\right)\left(5\sqrt{3}+3\sqrt{2}\right)=$

$$=4\sqrt{3}\cdot 5\sqrt{3}-3\sqrt{2}\cdot 5\sqrt{3}+4\sqrt{3}\cdot 3\sqrt{2}-3\sqrt{2}\cdot 3\sqrt{2}=$$
$$=20\sqrt{9}-15\sqrt{6}+12\sqrt{6}-9\sqrt{4}=$$
$$=20\cdot 3-3\sqrt{6}-9\cdot 2=$$
$$=60-3\sqrt{6}-18\boxed{=42-3\sqrt{6}}$$

17. $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$

Solution: Multiply the denominator and the numerator of the irrational expression with the binomial CONJUGATE to the polynomial in the denominator, NOT the same!!!

$$\frac{\sqrt{5}-1}{\sqrt{5}+1}\cdot\frac{\sqrt{5}-1}{\sqrt{5}-1}=\frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}-1\right)}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)}=\frac{\sqrt{5}\cdot\sqrt{5}-1\sqrt{5}+\sqrt{5}\cdot(-1)-1(-1)}{\sqrt{5}\cdot\sqrt{5}+1\sqrt{5}+\sqrt{5}\cdot(-1)+1(-1)}=$$
$$=\frac{\sqrt{25}-\sqrt{5}-\sqrt{5}+1}{\sqrt{25}\cancel{+\sqrt{5}}\cancel{-\sqrt{5}}-1}=\frac{5-2\sqrt{5}+1}{5-1}=\frac{6-2\sqrt{5}}{4}$$

REMEMBER to simplify the numerator and the denominator by dividing EVERY SINGLE TERM by 2. The solution is:

$$\frac{3-\sqrt{5}}{2}=\boxed{\frac{3}{2}-\frac{\sqrt{5}}{2}}$$

18. $\sqrt{\dfrac{16}{3}}$

Solution: Multiply both the numerator and the denominator inside the square root by 3, so you could simplify the denominator:

$$\sqrt{\frac{16}{3}}=\sqrt{\frac{16\cdot 3}{3\cdot 3}}=\sqrt{\frac{4^2\cdot 3}{3^2}}=\boxed{\frac{4\sqrt{3}}{3}}$$

In 19-20, solve and check each solution.

19. $\sqrt{2x+1}+5=8$

Solution: Solve for the square root: $\sqrt{2x+1}+5-5=8-5$

$\sqrt{2x+1}=3$

Square both sides of the equation:

$\left(\sqrt{2x+1}\right)^2=3^2$

$2x+1=9$

Solve for x:

$2x+1-1=9-1$

$2x=8$

$x=4$

DO NOT FORGET to check your answer by plug in $x = 4$ into the very first equation!

$\sqrt{2(4)+1}+5=8$

$\sqrt{9}+5=8$

$3+5=8 \qquad correct!$

The solution is: $\boxed{x = 4}$

20. $\sqrt[3]{4-2x}=2$

Solution: Power both sides of the equation by 3: $\left(\sqrt[3]{4-2x}\right)^3=2^3;\ 4-2x=8$

Solve by using the proper operations.

$4-2x-4=8-4$

$-2x=4$

$x=-2$

Do not forget to check your solution!!!!

$\sqrt[3]{4-2(-2)}=2$

$\sqrt[3]{4+4}=2$

$\sqrt[3]{8}=2 \qquad correct!$

The solution is: $\boxed{x=-2}$

21. Find the distance between the point (-5,8) and the point (1, 16).

Solution: The distance formula is: $d=\sqrt{\left(x_1-x_2\right)^2+\left(y_1-y_2\right)^2}$

$x_1=-5; \quad x_2=1; \quad y_1=8; \quad y_2=16$

$d=\sqrt{\left(-5-1\right)^2+\left(8-16\right)^2}$

Don't forget to use parentheses properly!!!

$d=\sqrt{\left(-6\right)^2+\left(-8\right)^2}$

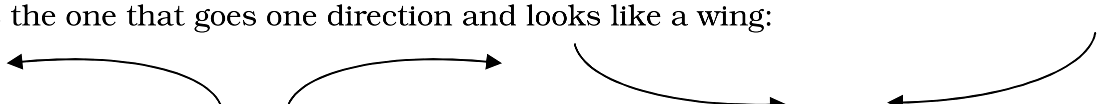
$d=\sqrt{36+64}=\sqrt{100}=10$

The answer is 10 units.

22. Graph the following radical function and list its domain and range. Label at least 3 exact points on your graph.

$f(x)=\sqrt{x+1}$

Solution: First locate the corner of the graph. This is the value for x, that makes the radicand (the binomial under the root) equal to 0. REMEMBER: the square root function is the one that goes one direction and looks like a wing:

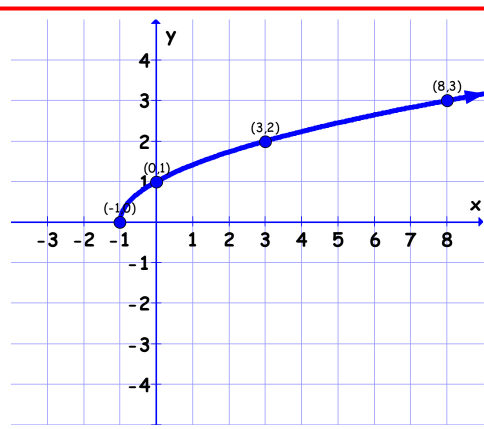


Once you calculate the corner point you have to calculate at least 2 other points:
For the function above, x = -1 makes the function 0, so the corner point is (-1, 0).

You cannot go to left, because when x = -2, $f(x)=\sqrt{-2+1}=\sqrt{-1}$, which does not exist and cannot be graphed. So you pick points to the right of x = -1. For example x = 0, x = 1… Some y values are not exact squares, so you avoid graphing them. Keep looking for exact y values to be able to graph exact values on the graph. In this example good points for graphing are: (-1, 0), (0, 1), (3, 2), (8, 3).

Domain: $[-1,\infty)$

Range: $[0,\infty)$



23. Solve by completing the square: $x^2-4x-1=0$

Solution: To complete the square first add 1 to both sides. Leave only multiples of x to the one side of the equation: $x^2-4x=1$

Then add $\left(\frac{-4}{2}\right)^2$ to both sides of the equation. (This is the coefficient of the term containing *x*, divided by *2* and squared.)

$$x^2-4x+\left(\frac{-4}{2}\right)^2=1+\left(\frac{-4}{2}\right)^2$$

$$x^2-4x+(-2)^2=1+(-2)^2$$

$$(x-2)^2=1+4$$

$$(x-2)^2=5$$

Use the square root property (take square root of both sides) to solve for x:

$$\sqrt{(x-2)^2}=\sqrt{5}$$

$$x-2=\pm\sqrt{5}$$

$$\boxed{x=2\pm\sqrt{5}}$$

24. Solve using the quadratic formula: $x^2-6x=-6$

Solution: Make sure that all the terms are to the one side of the equation!

$$x^2-6x+6=-6+6$$

$$x^2-6x+6=0$$

The quadratic formula is:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$a=1,\ b=-6,\ c=6$$

$$x=\frac{-(-6)\pm\sqrt{(-6)^2-4(1)(6)}}{2(1)}$$

$$x=\frac{6\pm\sqrt{36-24}}{2}=\frac{6\pm\sqrt{12}}{2}=\frac{6\pm2\sqrt{3}}{2}=\boxed{3\pm\sqrt{3}}$$

Don't forget to cancel the 2 from every single term of the expression!!!

In 25-27, perform the indicated operations. Give all answers in a + bi form.

25. $(6-\sqrt{-36})-(4+\sqrt{-49})$

Solution: The very first step is to remove the "-" sign from the radicals and to replace it with an "i" OUTSIDE of the radicals!

$(6-i\sqrt{36})-(4+i\sqrt{49})$

Simplify (if it is possible. FOLLOW the order of operation! There is a "-" sign to be distributed to the second parentheses!

$6-i\sqrt{36}-4-i\sqrt{49}=6-6i-4-7i=\boxed{2-13i}$

26. $(5-3i)(4-2i)$

Solution: FOIL and simplify. Do not forget that $i^2=-1$:

$5(4)-3i(4)+5(-2i)-3i(-2i)=$

$20-12i-10i+6i^2=$

$20-22i-6=$

$\boxed{14-22i}$

27. $\dfrac{2-i}{3+i}$

Solution: This question is very similar to question 18. Multiply the denominator and the numerator of the complex expression with the CONJUGATE to the denominator, NOT the same number!!!

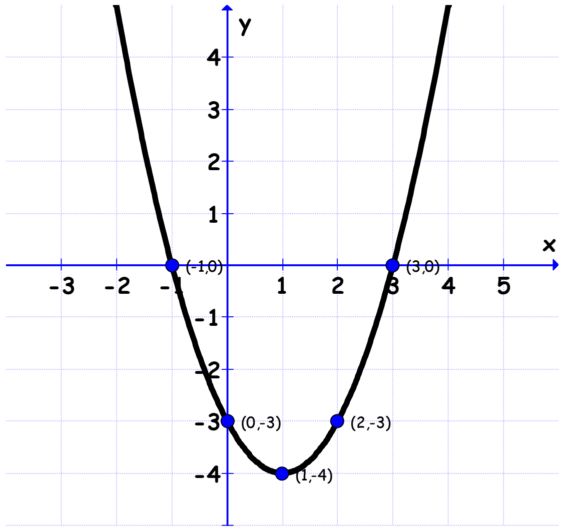
$$\frac{2-i}{3+i}\cdot\frac{3-i}{3-i}=\frac{(2-i)(3-i)}{(3+i)(3-i)}=\frac{2\cdot3-i\cdot3+2\cdot(-i)-i(-i)}{3\cdot3+i\cdot3+3\cdot(-i)+i(-i)}=$$

$$=\frac{6-3i-2i+i^2}{9\cancel{+3i}\cancel{-3i}-i^2}=\frac{6-5i-1}{9+1}=\frac{5-5i}{10}=\frac{5}{10}-\frac{5i}{10}=\boxed{\frac{1}{2}-\frac{1}{2}i}$$

28. Graph the given quadratic function. Label the vertex, axis of symmetry and 2 other exact points. List domain and range.

(a) $f(x) = x^2 - 2x - 3$

Solution: Find the vertex and calculate 2 points to the left and 2 points to the right.



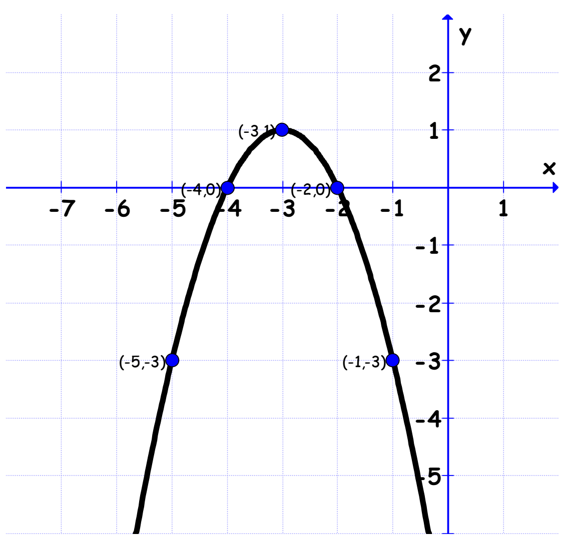
$$x_{vertex} = -\frac{b}{2a} = -\frac{-2}{2(1)} = \frac{2}{2} = 1$$

$$y_{vertex} = f(1) = (1)^2 - 2(1) - 3 =$$
$$= 1 - 2 - 3 = -4$$

Domain: $(-\infty, +\infty)$; Range: $[-4, \infty)$

(b) $f(x) = -(x+3)^2 + 1$

Solution: To find the points, first find the value for x at the vertex and then pick 2 values to the right and to values to the left. Substitute in the function and calculate the y value at these points. The vertex for a equation $f(x) = (x-h)^2 + k$ is (h,k), so for this equation it is at $(-3,1)$.



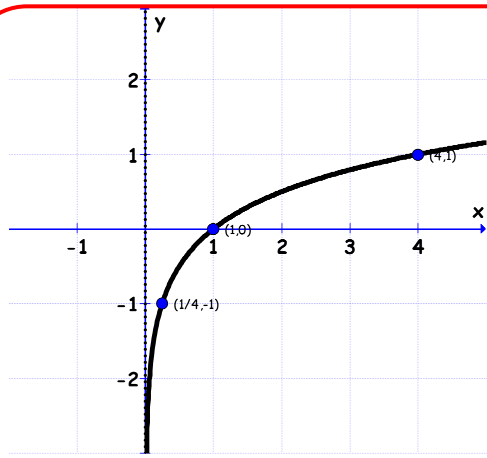
Domain: $(-\infty, +\infty)$; Range: $(-\infty, 1]$

29. Graph the function $f(x)=\log_4 x$. Label 3 exact points. List domain and range.

Solution: The easier points to be plugged are:

$$(1,\log_4 1)=(1,0);\quad (4,\log_4 4)=(4,1);\quad \left(\frac{1}{4},\log_4\frac{1}{4}\right)=\left(\frac{1}{4},-1\right)$$

Pay attention! There is a vertical asymptote at x = 0! The function inside of the log is ALWAYS greater than 0.



Domain: $(0,+\infty)$; Range: $(-\infty,\infty)$

30. Graph the function $f(x)=3^x$. Label 3 exact points. List domain and range.

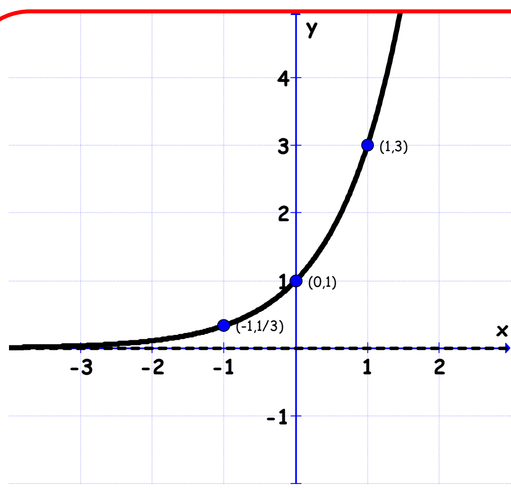
Solution: The easier points to be plugged are:

$$(0,3^0)=(0,1);$$

$$(1,3^1)=(1,3);$$

$$(-1,3^{-1})=\left(-1,\frac{1}{3}\right).$$

Pay attention! There is a horizontal asymptote at y = 0! 3^x cannot be less than 0.



Domain: $(-\infty,+\infty)$; Range: $(0,\infty)$

In 31-33, solve for x.

31. $\log_x 81 = 4$

Solution: Every time when you cannot plug in the calculator, use the DEFINITION for logarithm to switch to exponential form or vice versa!

$\log_x 81 = 4 \quad \Leftrightarrow \quad x^4 = 81$

$x^4 = 3^4$

$\boxed{x = 3}$

Don't forget to plug back in the very first equation to see if x = 3 is solution:

$\log_3 81 = 4$

$\log_3 3^4 = 4 \quad \textit{correct}!$

32. $\log_{10} 100 = x$

Solution: $\log_{10} 100 = x \quad \Leftrightarrow \quad 10^x = 100$

$10^x = 10^2$

$\boxed{x = 2}$

33. $\log_2 x = 4$

Solution: $\log_2 x = 4 \quad \Leftrightarrow \quad 2^4 = x; \qquad \boxed{x = 16}$

34. Write the expression $\log\ a^6 b^2 c$ in terms of the logarithms of a, b and c.

Solution: $\log\ a^6 b^2 c\ = \log\ a^6 + \log\ b^2 + \log c\ = \boxed{6\log\ a + 2\log\ b + \log c}$

35. Write the expression $\frac{1}{3}\ln\ a + 7\ \ln\ b - \ln\ c$ as a logarithm of a single quantity.

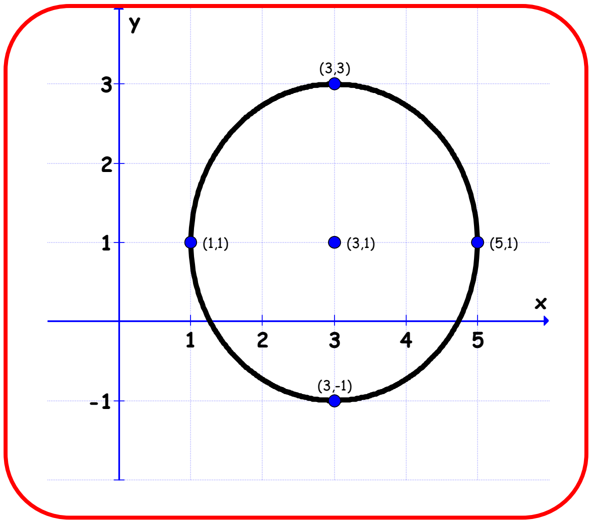
Solution: $\frac{1}{3}\ln\ a + 7\ \ln\ b - \ln\ c = \ln\ \sqrt[3]{a}\ + \ln\ b^7 - \ln\ c = \boxed{\ln\ \frac{\sqrt[3]{a}\ b^7}{c}}$

36. Use the change of base formula to find $\log_4 5$ to four decimal places.

Solution: $\log_4 5 = \frac{\log\ 5}{\log\ 4} = \frac{0.6989700043}{0.6020599913} \approx 1.16096 \approx \boxed{1.1610}$

37. Sketch a graph of the circle $(x-3)^2+(y-1)^2=4$. Label 4 exact points.

Solution: The center is at (3, 1). The radius is r = 2. Locate the center and count 2 units each direction.



38. On the same set of axes graph the one-to one function, $f(x)=-\frac{1}{4}x+1$ and its inverse.

Write the equation of f^{-1}.

Solution: To find the inverse function follow the steps:

1. Replace $f(x)$ with y. $y=-\frac{1}{4}x+1$

2. Switch the places of x and y. $x=-\frac{1}{4}y+1$

3. Solve for y. $(4)x=(4)\left(-\frac{1}{4}\right)y+(4)1$

$$4x=-y+4$$

$$\cancel{4x}\cancel{-4x}+y=\cancel{-y}+4-4x+\cancel{y}$$

$$y=-4x+4$$

4. Replace y with f^{-1}. $f^{-1}(x)=-4x+4$

Graph both lines by calculating at least 3 points from each of them. If you have worked correctly, the lines are going to be symmetrical about the line $y = x$.